New interactions of Combinatorics and Probability

CIMPA – Summerschool
AUGUST 24 – SEPTEMBER 4, 2015

ICMC, Universidade de São Paulo, Brazil

PRELIMINARY SCIENTIFIC PROGRAM
(Updated on August 19th 2015)

MINICOURSES

Introductory Lecture

Philippe Biane, CNRS - Univ. Paris Est, France
Roland Speicher, Univ. of Saarbrücken, Germany

(2 hrs) – The Introductory Lecture will open the School and give a broad overview of the history and current trends in the field of Interactions between Combinatorics and Probability.

Determinantal processes and combinatorics

Philippe Biane, CNRS - Univ. Paris Est, France

(7 hrs) – The lecture will survey the theory of determinantal processes and give some examples coming from combinatorics and representation theory.

Algebraic Combinatorics

Kurusch Ebrahimi-Fard, ICMAT-CSIC, Madrid, Spain
Anke Wiese, Heriot-Watt Univ., Edinburgh, Scotland

(4+1 hrs) – The seminal works of G.-C. Rota and his school, have transformed algebraic combinatorics into an important branch of mathematics with connections to a wide variety of subjects, among others, numerical methods for (partial/stochastic) differential equations; control theory; quantum field theory, (free) probability, number theory, discrete geometry, algebraic geometry. In these lectures we will introduce in a pedagogical way the basic notions and key results relevant in connection with applications in the main topics of this school.

Rough Paths Theory

David Kelly, Courant Institute, New York University, NY, USA

(6 hrs) – We explain the basic ideas behind T. Lyons' theory of rough paths. This theory provide a general framework in which it is possible to analyze the behavior of differential systems controlled by non-smooth signals, like for example, the paths of many interesting stochastic processes, such as Brownian motion and its fractional counterparts. The aim of the theory is to construct a solution map which sends an irregular signal x to the solution y of a differential equation of the form $dy(t) = f(y(t))dx(t)$. The key idea is that one must extract finer information from the signal x in the form of a more detailed object known as a 'rough path', this extra information is then used to construct the solution y in a systematic way. The space of rough paths itself contains rich algebraic and combinatorial structure akin to the Butcher series framework encountered in numerical analysis. If time permits, we will discuss very recent developments of rough path ideas and their algebraic counterparts in the realm of stochastic PDEs and renormalization.

Numerical integration, structure preservation and combinatorics

Hans Munthe-Kaas, Univ. of Bergen, Norway

(4 hrs) – Numerical integration of differential equations is a central topic in computational mathematics. A numerical integrator produces an approximation to the exact flow of the differential equation. Traditionally, order conditions and stability are the main design criteria of a numerical algorithm. More recently structure preservation has been recognized as an important qualitative feature. Much effort has been put into the design of algorithms which exactly preserve important geometric features such as first integrals, energy, impulse, continuous and discrete symmetries, symplectic and Hamiltonian structures as well as energy and volume preservation and intrinsic integration algorithms of systems evolving on manifolds. This gave rise to Geometric Integration as a research area in numerical analysis. For half a century B-series (after J. Butcher) has been one of the most important tools for analyzing numerical integration schemes. We have in the last two decades understood how various questions about structure preservation can be studied as algebraic conditions on the B-series expansion of the numerical integrator. More recently a comprehensive theory of integration of Lie groups and manifolds has been developed, and along with this a generalization of B-series to manifolds, so-called Lie-Butcher series. Whereas B-series is algebraically described in terms of pre-Lie algebras, the Lie-Butcher series are based on the recently defined concept of post-Lie algebras. These structures are closely related to invariant connections on manifolds.

We will in the talks give an extensive and self-contained introduction to the underlying algebraic structures and discuss how these relate to concepts in differential geometry and to various application areas.

Free Probability

Roland Speicher, Univ. of Saarbrücken, Germany

(6 hrs) – The theory of free probability is a relatively new branch of mathematics, which was started by D. Voiculescu in the early 1980s, when he realized that it could be useful to borrow techniques from probability theory to attack problems in the theory of operator algebras. This let him to define a new, non-commutative, probability, where the classical concept of independence is fruitfully replaced by the notion of freeness. In a nutshell, free probability is a non-commutative probability theory plus freeness. In this new context, most of the standard concepts of classical probability, such as the central limit theorem, entropy and Brownian motion, find their natural analogues. Free probability became not only useful in studying problems in operator algebra theory, but also as an independent subject, connected to various part of mathematics, for example, algebraic and enumerative combinatorics and random matrix theory.

Stochastic Calculus

Anke Wiese, Heriot-Watt Univ., Edinburgh, Scotland

(5 hrs) – This course will provide students with an introduction to the mathematical concepts of stochastic integral calculus and stochastic differential equations. It's integration by parts formula is then the starting point to investigate the structure of the algebra generated by iterated integrals. This algebraic structure has recently been explored to design new and efficient methods to solve stochastic systems numerically, and new results in this area will be presented in the course. The course will end with an outlook towards stochastic partial differential equations. The overall aim is to introduce students to stochastic calculus and to the structure of the algebra of multiple stochastic integrals, and to show how the algebraic structure facilitates the design and analysis of stochastic numerical integrators.

CONFERENCES

Title TBA

Rodrigo Bissacot, IME - Universidade de So Paulo, So Paulo, Brazil

(1 h) – TBA.

Horton self-similarity of Kingman's coalescent

Yevgeniy Kovchegov, Mathematical Department - Oregon State University

(1 h) – We establish Horton self-similarity for a tree representation of Kingman's coalescent process. The proof is based on a Smoluchowski-type system of ordinary differential equations for the number of branches of a given Horton-Strahler order in a tree that represents Kingman?s N-coalescent process with a constant kernel, in a hydrodynamic limit. We also demonstrate a close connection between the combinatorial Kingman's tree and the combinatorial level set tree of a white noise, which implies Horton self-similarity for the latter. Numerical experiments illustrate the results and suggest that Kingman's coalescent and a white noise also obey a stronger Tokunaga self-similarity. This presentation is based on joint work with Ilya Zaliapin (University of Nevada Reno).

Bifurcations of random dynamical systems

Jeroen Lamb, Imperial College London, UK

(1 h) – We discuss early developments of a bifurcation theory for dynamical systems with noise. The ideas will be illustrated in the context of a specific example of a pitchfork bifurcation with additive noise. This is joint work with Mark Callaway, Martin Rasmussen, Doan Thai Son (Imperial College) and Christian Rodrigues (MPI Leipzig).

Bifurcations of mutually coupled equations in random graphs

Tiago Pereira, ICMC - Universidade de So Paulo, So Carlos, Brazil

(1 h) – We study the behavior of solutions of mutually coupled equations in heterogeneous random graphs. Heterogeneity means that some equations receive many inputs whereas most of the equations are given only with a few connections. Starting from a situation where the isolated equations are unstable, we prove that a heterogeneous interaction structure leads to the appearance of stable subspaces of solutions. Moreover, we show that, for certain classes of heterogeneous networks, increasing the strength of interaction leads to a cascade of bifurcations. This is a joint work with Eduardo Garibaldi.

Decomposition of stochastic flows in dual foliated manifold: extensions of time

P. R. C. Ruffino, IMECC - Universidade Estadual de Campinas, Campinas, SP, Brazil

(1 h) – Given two complementary foliations in a differentiable manifold say horizontal and vertical, consider the Lie subgroups of horizontal and vertical diffeomorphisms in M which preserve the corresponding leaves. There exists a factorization, up to a stopping time, of an stochastic flow of diffeomorphisms associated to an SDE on M into a diffusion in the horizontal subgroup composed with a process in the vertical subgroup.This decomposition has a natural geometric and dynamical interest e.g. on perturbation of a Hamiltonian system (constant energy leaves), coordinates of stochastic processes, horizontal lift of processes on fibre bundles, and others. (Catuogno, da Silva and Ruffino, Stoch. & Dyn. 2013). In this talk we are going to present extensions of time of this factorization. Essentially jumping diffeomorphisms which are not decomposable. The results open possibilities to study asymptotic properties. This is a joint work with Leandro Morgado.

Universality in discrete copulas and Brownian bridges

N. Saldanha, Pontificia Universidade Católica do Rio de Janeiro, Brazil

(1 h) – Copulas can be discretized in more than one way, yielding objects which for appropriate scaling limits give rise to Brownian bridges. Joint work with Juliana Freire and Carlos Tomei (both from PUC-Rio).

Random walks in a queueing network environment

Anatoly Yambartsev, IME - Universidade de So Paulo, So Paulo, Brazil

(1 h) – We propose a class of models of random walks in a random environment where an exact solution can be given for a stationary distribution. The tool is the detailed balance equations. This is joint work with E. Pechersky, Y. Suhov and M. Gannon.

Closing talk: Invariants of group actions on free structures

Claude Cibils, CIMPA / Université de Montpellier, France

(1 h) – In this talk I will give first a presentation of actions of finite groups on commutative structures and results about invariants. Then I will concentrate on non-commutative free algebras over a field, and on free categories over a field.

POSTERS

A spatial stochastic model for a two-stages innovation diffusion

Karina B. E. Oliveira, ICMC - Universidade de São Paulo / Universidade Federal de Sõ Carlos, Brazil

We propose and study a spatial stochastic model describing a process of awareness, evaluation and decision-making by agents on the d-dimensional integer lattice. Each agent may be in any of the three states belonging to the set $\{0, 1, 2\}$. In this model 0 stands for ignorants, 1 for aware and 2 for adopters. Aware and adopters tell about a new product innovation to any of its (nearest) ignorant neighbors at rate λ. At rate α an aware becomes an adopter due to the influence of (nearest) adopters neighbors. Finally, aware and adopters forget the information about the new product at rate one. The purpose of this work is to analyze the influence of the parameters on the qualitative behavior of the process. More precisely, we study sufficient conditions under which the innovation diffusion (and adoption) either becomes extinct or propagates through the population with positive probability. This is a joint work with Cristian Coletti (UFABC) and Pablo Rodriguez (ICMC-USP).

Percolation on random causal triangulation

José Javier Cerda Hernández, IMECC, Universidade Estadual de Campinas, Brazil

In this work we study bond percolation on random causal triangulation. We show that the phase transition is non-trivial and we compute bounds for the critical value. Obviously, the critical value depends strongly on the nature of the underlying graph, but the critical value is shown to be constant a.s.

Invariance under quasi-isometries of sub and supercritical behavior in Boolean percolation

Cristian Coletti, CMCC, Universidade Federal do ABC, Brazil

In this work we study the Poisson Boolean model of percolation in locally compact Polish metric spaces and we prove the invariance of subcritical and supercritical phases under mm-quasi-isometries. More precisely, we prove that if a metric space M is mm-quasi-isometric to another metric space N, and the Poisson Boolean model in M exhibits any of the following: (a) a subcritical phase; (b) a supercritical phase; or (c) a phase transition, then respectively so does the Poisson Boolean model of percolation in N. Then we use these results in order to understand the phase transition phenomenon in a large family of metric spaces. Indeed, we study the Poisson Boolean model of percolation in the context of Riemannian manifolds, in a large family of nilpotent Lie groups and in Cayley graphs. Also, we prove the existence of a subcritical phase in Gromov spaces with bounded growth at some scale.

Applying Finite Elements method to Kardar-Parisi-Zhang (KPZ) equation

Ciro Javier Díaz Penedo, IMECC, Universidade Estadual de Campinas, Brazil

In this work we studied numerical approximation of solution of the KPZ equation. The discretization methodology used was Mixed and Hybrid Finite Element Method with domain decomposition. The system of algebraic equations obtained at each time step was solved using an iterative fixed point algorithm. We present some numerical results to illustrate the performance of this methodology on approximating the solution of the KPZ in one spatial dimension.

An Euler-Maruyama-type approach for the Cox-Ingersoll-Ross process

Ricardo Ferreira, ICMC - Universidade de São Paulo / Universidade Federal de São Carlos, Brazi

The Cox-Ingersoll-Ross process was originally proposed by John C. Cox, Jonathan E. Ingersoll Jr. and Stephen A. Ross in 1985. Nowadays, this process is widely used in financial modelling, e.g. as a model for short-time interest rates or as volatility process in the Heston model. The stochastic differential equation (SDE) which defines this model does not have closed form solution, so we need to approximate the process by a numerical method. In the literature, several numerical approximations has been proposed based in interval discretization. In this work, we approximate the CIR process by Euler-Maruyama-type method based in random discretization proposed by Leo e Ohashi (2013) under Feller condition. In this context, we obtain an exponential convergence order for this approximation and we use Monte Carlo techniques to compare the numerical results with theoretical values.

Persistence of Network Synchronization: A Non-Identical Coupling Functions Model

Marcos Daniel Nogueira Maia, Instituto Nacional de Pesquisas Espaciais - INPE, Brazil

We investigate the persistence of the synchronization in networks of coupled oscillators when the coupling functions are non identical. Under mild conditions, we prove that nonlinear mismatches in the coupling function have no effect at synchronization, whereas linear mismatches, if small enough, can decrease the speed of the convergence towards synchronization. The results provides sufficient conditions for synchronization's persistence in terms of the network structure.

One-Baryon Spectrum and Analytical Properties of One-Baryon Dispersion Curves in $3+1$ Dimensional Strongly Coupled Lattice QCD With Three Flavors

José Carlos Valencia Alvites, ICMC, Universidade de So Paulo, So Carlos, Brazil

Considering a $3+1$ dimensional lattice QCD model defined with the improved Wilson action, three flavors and 4×4 Dirac spin matrices, in the strong coupling regime, we reanalyze the question about the existence of the eightfold way baryons and complete our previous work where the existence of isospin octet baryons was rigorously solved. Here, we show the existence of isospin decuplet baryons which are associated with isolated dispersion curves in the subspace of the underlying quantum mechanical Hilbert space with vectors constructed with an odd number of fermion and antifermion basic quark and antiquark fields. Moreover, smoothness properties for these curves are obtained. The present work deals with a case for which the traditional method to solve the implicit equation for the dispersion curves, based on the use of the analytic implicit function theorem, cannot be applied. We do not have only one but two solutions for each decuplet sector with fixed spin third component and, instead, we use the Weierstrass preparation theorem. This work is completed by establishing and analyzing a spectral representation for the two-baryon correlation and providing the leading behaviors of the field strength normalization and the mass of the spectral contributions with more than one-particle. Joint work with Paulo A. Faria da Veiga and Michael O'Carroll (ICMC, USP-São Carlos).

Connections in Generalized Symplectic Manifolds with Lagrangian Foliation

Sandra Zapata Yepes, CMCC, Universidade Federal do ABC, Brazil

This work is naturally divided into two parts. In the first part, we present results on the geometry of symplectic manifolds equipped with a given lagrangian foliation: the construction of the (partial) Bott connection, the classification of (full) connections compatible with the given structures, namely, the symplectic form as well as the foliation, and Weinstein's tubular neighborhood theorem for (lagrangian) sub manifolds transversal to the foliation. Most of these results are well known, but are here formulated in such a way as to allow their extension to a more general setting. In the second part, we study such an extension, namely in the context of the geometry of poly lagrangian and multilagrangian (in particular, polysymplectic and multisymplectic) fiber bundles, which come with a canonical (poly- or multi-)lagrangian distribution. These structures are important since they are central tools in the covariant hamiltonian formalism of classical field theory.

Optimal synchronization of Kuramoto oscillators: a dimensional reduction approach

Rafael Soares Pinto, Universidade Estadual de Campinas

The recently proposed dimensional reduction approach for the Kuramoto model [G. Gotwald, Chaos 25, 053111 (2015)] is employed to construct optimal network topologies to favor synchronization. Some explicit analytical conditions matching previous know results are obtained. Joint work with A. Saa.